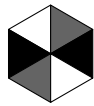
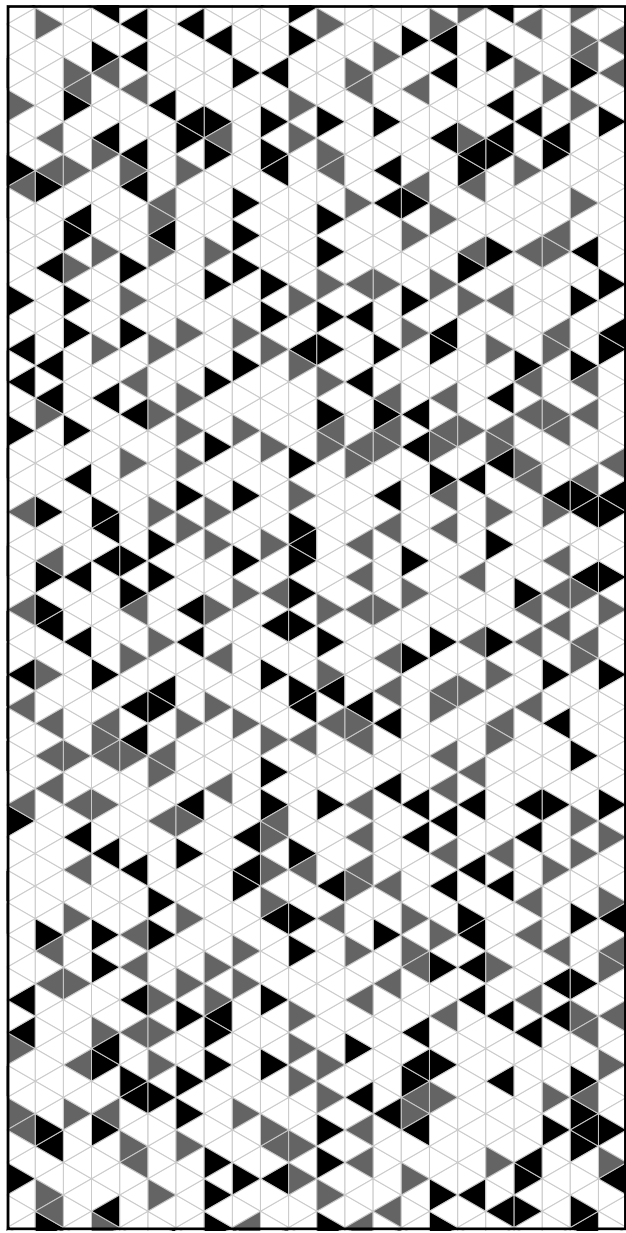
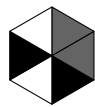


**Place your bets!**



**Using maths to give you the edge**

# Place your bets!



?/!

## Did you say luck?

Play heads or tails with a friend. Your opponent wins three times in a row. That's strange? Ten times! Isn't that just a little suspicious?

You meet your neighbour in China. Is that a surprise?

Five planes crash in just one month. Should you cancel your holiday?

When should we talk about coincidences? And when should we look for a deeper explanation?

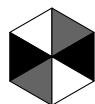
Chance isn't always unpredictable. Using maths, we can create laws that govern random events. Using these, we can improve our intuition and avoid falling into superstition... Did you know that it is common for a class to have two pupils with the same birthday? That the weather forecast for the next two weeks is uncertain, but we can predict what the climate will be in 15 years?

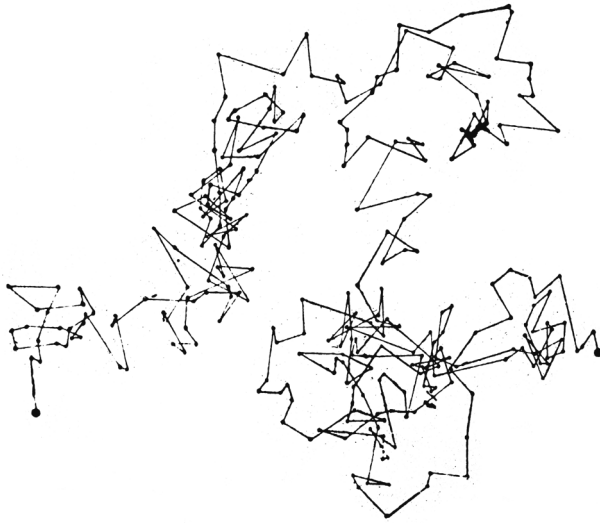
This exhibition will challenge how you look at chance and luck, offering you the keys to understand it and use it to your advantage.

Now it's time to have some fun. Ready?  
Now place your bets!

!/!

Using maths to give you the edge





*"Brownian motion" is completely random: each time, the point can move in any direction. It can be used to model the movement of a particle on water, like in the example above. This was an historical observation, made by Jean Perrin.*

3

## Three computer programmes



**"The march of the drunkard"**: at each step, the point has the same chance of moving to any of the four directions (left, right, up and down).

**"Exponential decrease"**: at each step, each white (full) box has one chance in two of becoming empty.

**"A random Sierpinski triangle"**: at each step, add a centre point between the last point you've put in and one of the three points of the triangle.

A random distribution of black and white points appears. Is this what you thought random chance looked like?



Three computer programmes



# Probabilities

?!/i

**Let's roll a die.**

You've got a one in six chance of rolling a 4, and a one in two chance of rolling an even number. To calculate these probabilities, you need to divide the number of positive outcomes (what you're hoping will happen) by the total number of possibilities.

But what if the die is loaded, so it rolls one number more often than the others? The system is no longer **equiprobable**, so we need to adapt our method!

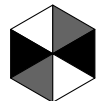
**Another key idea: independence.**

You can roll a 6 on one throw, but this won't have any effect on the next one. However, this is not the case for everything - whether you have a driver's license or not, for example, is not independent of where you live...

**Sometimes you can evaluate probabilities using past experience.** But the number of outcomes you examine has to be very large. If a lift breaks down 5 times in ten years, then every year there's one chance in two that it will break down!

!/?i

# Probabilities



## The second always comes first



Player 1 chooses a die. Player 2 then chooses a different one, from the three that remain. **Roll the dice!** The player with the highest score wins a point. **Start again!**

Whichever die is chosen by player 1, player 2 can always choose a die with a greater chance of winning.

Try and understand why...



Blue die against yellow die: If the blue die rolls a 5, it wins; if it rolls a 1, it wins two times out of six. Overall, it wins more often than it loses.

Similarly, you can show that the yellow die beats the green die; the green die beats the red die and the red die beats the blue die!

**Not very intuitive, is it?**

*These dice were invented by the American statistician Bradley Efron (born in 1938).*



The second always comes first

5

## The dice snake



**Roll** all the dice, then **line them up** into a snake. **Read** the number on the first die. **Move** forward along the snake by that number of dice. **Read** the number of the die you've arrived at. **Move** forward by that number once again. **Keep** going until there aren't enough dice left to move forward by the number you arrive at. **Remove** the remaining dice ahead of you.

Now **take** the first die in the line (and only the first). **Roll** it, **put it back** where it was, and **start** the game again.

What do you notice?



**Did you end up in the same place both times?**

When you play a second time, if you land on any of the dice you landed on during the first game then your path forward will be identical.

**This means that there's a much greater than one-in-six chance of ending up at the same place.**



The dice snake

## Red dice



**Roll** all the dice at the same time. For all the dice which rolled red side up, **line** them up in the first column. **Roll** the remaining dice. Line up the reds in the second column. **Repeat** this until there are no more dice to roll.

**What can you see?**



A radioactive nucleus has a certain probability of decaying, and this probability is always the same. Half of the nuclei will decay in a given amount of time. It will take roughly the same amount of time to decay half of the remaining nuclei. And it goes on like this: the fewer nuclei there are, the slower they decay.

**This process is “exponential”, just like our dice.**



Red dice

6

## 1 in 10,000



This glass tube contains 10,000 marbles.

9,999 of them are blue, and just one is white.

**Do you think** you have any chance of spotting it?

**Slowly turn the tube and have a look!**



As a comparison, **you’ve got one chance in... 19 million** to win the jackpot in the lottery!



1 in 10,000

**Place** the panel on the random pattern so that the red edges line up exactly.

A shape appears.


**Turn** the panel half-way around, and you'll see another shape.



Shapes in the fog

## Shapes in the fog



 One of these panels is randomly covered in black or transparent squares. On the second, we've blacked out the boxes that make the shape, which match up to the empty boxes on the first panel.

**The patterns that this makes may look completely random, but when you put the panels together, the picture appears in black.**

*"Visual cryptography" was invented in 1994 by the Israeli mathematicians Moni Naor and Adi Shamir.*

7

**Turn** the panel until the visible letters make a sentence that makes sense.

Not so easy, is it? **Turn it again.**


**How many sentences can you find?**



Hidden message

## Hidden message



 Like with "Shapes in the fog", this experiment will show you one of the uses of chance: **introduce confusion!** It's used in a lot of classic and modern encryption methods.

**Finding a pattern among a lot of signs placed at random is really hard!**

## Roulette



You've been given some counters.

Make a **"double bet"**: one chip on red or black, the other on odd or even.

**Spin** the roulette wheel. Did you get the colour or number you were hoping for?

**Do all double bets give you the same chance of winning?**



You've got the same probability of spinning red or black, odd or even. It's less than 50% because of the 0 and 00.

**But double bets don't always give you the same chance!** For example, there are more boxes that let you win with "black and even" than with "black and odd".

**One thing is sure though: the casino will always win in the end!**



Roulette

8

## The number collector



**Place all** of the numbered squares at the top of the board.

**Roll** the dice.

**Move** a square down towards you when you roll the number on the square the first time.

**How many throws do you need to bring all of the squares down?**



The last stickers in a collection always seem the most difficult to get – it's as if they are rarer than the others. The explanation for this is simple: the more stickers you have, the greater the chance you have of finding a double.

**Here, it works the same way. The last number takes a long time to bring down:** on average, you'll need to throw 15 times before you collect them all.



The number collector



## Loaded dice



Here are two pairs of dice, one red and one black. In each, one of the dice is loaded.

**Just by rolling the dice a few times, can you tell which dice are loaded?**



If you roll a fair die six times, the chances of it landing once on each side are pretty slim!

**This makes it difficult to catch a loaded die quickly.** The only way to catch it out is to throw it many, many times.



Loaded dice

9

## Minipoly



The rules are simple: **roll** the die and **move** your counter forward the number of spaces given by the die. Play for a while.

Where do you land more often: Champs-Élysées or jail?

Keep track with the abacus.



On real Monopoly<sup>®</sup> too, not every street offers the same probability that you'll land on it.

**This is mostly because of the "Go to Jail" square.**



Minipoly

## Not the same colours!



Shake the box.

Look at the colours that the marbles are sitting on.

Can you make it so that no marble is sitting on its own colour?



For 2 marbles and 2 colour, there are only 2 possible positions: this means there is a one-in-two chance that at least one marble (in fact, both of them) is on its own colour. For 3 marbles and 3 colours, the probability increases to 2 in 3.

And the more marbles you use, the more the probability homes in on a value a touch below  $2/3$ .



Not the same colours!

10

## Sicherman dice



What are the different totals you can get by rolling these two dice?

Are some more likely than others?

Compare these probabilities with the probabilities you get with normal dice!



By rolling both dice, you can get every value from 2 to 12, just like with normal dice. What's even more surprising is that the probability of rolling these totals is the same, too! **Check this with the tables.**

These dice were invented by **George Sicherman** and are the only dice with this property.



Sicherman dice

## Blue, yellow, green



Set the counters to zero (slide all the rings to the left). Roll the dice. If both come up blue, slide one blue ring to the right. If both are yellow, slide a yellow ring. If they come up different colours, slide a green ring.

Roll the dice again a few times.

Do all three counters increase at the same speed?



The green counter advances almost twice as quickly!  
**Why does it do this?**

**Look at this table:** the green turns up twice as often as each of the two other colours!

		Die 1	
		Blue	Yellow
Die 2	Blue	Blue	Green
	Yellow	Green	Yellow



Blue, yellow, green



Probabilities

"A roll of the dice will never abolish chance"

Stéphane Mallarmé

Probabilities



Probabilities

"The lottery is a tax on people who doesn't understand probabilities"

Anonyme

Probabilities



# Law of large numbers



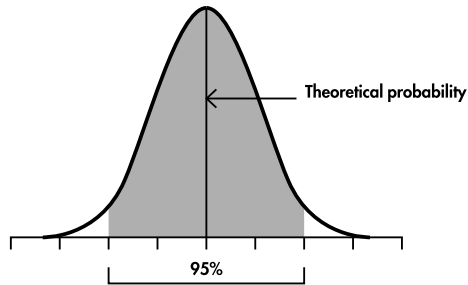
?!/i

You can't predict what will happen when you toss one coin.

But what about a thousand coins? Common sense would say that around 50% of the coins will land on heads - and so does the "law of large numbers".

This law states that, by repeating a random experiment a large number of times, the frequency of a given outcome will always approach the theoretical probability of it occurring.

The "central limit theorem" lets us state (with some accuracy) to what extent it is or isn't probable that an occurrence will be far from the theoretical probability when the number of throws is very large. In layman's terms: if the percentage of "heads" that you toss is a long way away from the expected 50%, then you should take a closer look at your coin!



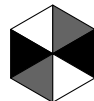
*The frequency observed has a 95% chance of falling into this range.*

*The central limit theorem: the difference between the theoretical probability and the frequency observed works according to a common law, often known as the "bell curve".*

© Guillaume Reuiller/Universcience

!/?/i

# Law of large numbers



## Break the code!



On this screen, you can read a coded piece of text.

**Try** and **guess** which letter should replace each coded letter, and write it below the letter. To do this, **use** the keyboard and the arrows.

Hint: **start** with the shorter words; the most frequently used coded letter might correspond with the one that turns up most often in a piece of French text...



A “frequency analysis” enables you to decipher a message, as long as each letter is always encrypted in the same way.

It uses our own statistical knowledge of the language we use: the frequency of each letter, of each group of two letters (digraph) or three letters, the most frequently occurring words, etc.



Break the code!

13

## Galton's box



**Take** a marble and drop it in the hole.

**Follow** the path it takes.

**Drop** another one. Do the marbles all have the same chance of dropping into each box? How many paths lead to the box furthest on the right? And the box next to it?

*When there are no more marbles, please press both levers.*



By dropping a lot of marbles, you can experiment the law of large numbers and see for yourself that the result always resembles the same “bell curve”. This is everywhere in statistics.

To find out more about how many paths lead to a specific box, have a look at Pascal's triangle.

*Galton's box was invented by the English statistician, Sir Francis Galton (1822-1911).*



Galton's box

## Numbers on the front page



Here are two pages from a newspaper. You can see some numbers on them. On the left-hand page, **place** a red counter on the numbers which begin with a 1, and a yellow counter on those which begin with a 2. On the right-hand page, **place** a red counter on the numbers which begin with a 1 and a yellow counter on those which begin with a 9.

**Are there as many red counters as yellow counters? And as many yellow counters on the left as on the right?**



This phenomenon is known to mathematicians under the name “Benford’s law”.

Benford’s law is counter-intuitive: in terms of numbers used in real life, a small number is always more frequent when used as the first digit than a larger number. One of the simplest explanations for this applies to the summits of the Alps: for altitudes greater than 1000m, only the numbers 1, 2, 3 and 4 are used!



Numbers on the front page

14

## The Monte Carlo method



**Shake** a box.

Each of the 100 counters will be arranged at random.

**Count** those which are sitting inside the shape drawn.

**Multiply** this number by 4 and you’ll have an estimate of the surface area of the shape in centimetres squared!



If you mixed them well, the quantity of counters spread over a shape will be proportional to its surface. The square measures  $400\text{cm}^2$  for 100 counters. If you find 78 counters in the circle, the rule of three gives  $78 \times 400$  divided by  $100 = 312$ . And the surface of the circle is approximately...  **$314\text{cm}^2$** .

*When we talk about the “Monte Carlo method”, we’re referring to the casino because this method uses chance to make a calculation.*



The Monte Carlo method

## Two Galton's boxes in a row



Spin the wheel and **observe** the number of marbles which fall on one side or the other...

On one of the boxes, the area in which the marbles bounce isn't centred perfectly.

Which one is it?



With a single marble, you would need to repeat the experiment a great many times to establish a trend. By using a large number of marbles, we can get an answer immediately and even a fairly accurate idea of the probability that each ball will go to one side or the other.

This is why we like to use the law of large numbers: **we can go from the idea of probability to the idea of proportion.**



Two Galton's boxes in a row

15



Law of large numbers

"Luck is only fickle when you look at individual outcomes."

Michel Serres et  
Nayla Farouki

Le Trésor,  
"The Law of Large  
Numbers"

Law of large numbers





# Counting

16

!/?

To calculate a probability, you need to count all the positive and possible outcomes – without making any mistakes. It's not always easy!

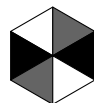
If you play heads or tails ten times, each throw will have two possible outcomes with no link to the previous throws. This gives  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$  possible results.

Shuffle a deck of 32 cards. The first card (one of the 32) is followed, at random, by one of the 31 remaining cards. The second, by one of 30. The number of possible orders is therefore  $32 \times 31 \times 30 \times 29$  [x etc.] x 1.

How many teams of 11 players can you make from a group of 22 people? This is harder to calculate! Now we're talking about "combinations". You'll find them on Pascal's triangle, not too far from here...

!/?

# Counting





## One hundred thousand billion poems



Turn each of the 14 rollers, each one with 10 lines printed on it, and compose your own sonnet!

**Raymond Queneau** wrote these lines so that any possible combination will work in terms of syntax and rhyme.

**Are there really 100,000 billion possible poems?**



**You are standing in front of a replica of the largest anthology of poetry\* ever written!**

For a 14-line sonnet, there are ten possible first lines. Then ten possible second lines, and so on, with each choice completely independent of the previous ones. This gives 10<sup>14</sup>, or a possible total of **100,000 billion poems!**

*\*Raymond Queneau, Cent mille milliards de poèmes © Éditions Gallimard, 1961*



One hundred thousand billion poems

17

## Let's play some music



**Compose** a unique piece of music by rolling dice. Each of the 16 bars of the melody will be chosen at random from a list of 176 bars according to a set process. Now **throw** both dice 16 times and **enter** the numbers you get into the computer. Or even **let** it choose the numbers itself at random!

The number of possible melodies is gigantic:

**759,499,667,166,482.**

**It's extremely likely that you'll be the first person to ever hear this piece of music!**



For this 16-bar melody, there are 11 possible first bars. Then 11 possible second bars, and so on, with each choice being independent of the choices made before.

This gives  $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$  ( $11^{16}$ ) – that is, unless Mozart didn't include any other little variations!

*This experiment was first conceived by Wolfgang Amadeus Mozart (1756-1791).*



Let's play some music

## Pascal's triangle



This famous triangle enables you to solve a number of different counting problems. **Look at it:** each number is the sum of the two numbers immediately above it.

**What is the link between this and Galton's box, just near you?**

**Turn the cubes with even numbers on them.**

**What do you see?**



On each cube, you'll see the number of paths leading to it from above, descending step by step from one cube to one of its two neighbours below.

**Doesn't this remind you of Galton's box?**

*A curious thing about this triangle: the even numbers are arranged according to the beginning of Sierpinski's triangle, a well-known fractal!*



Pascal's triangle



Counting

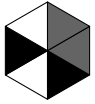
"The theory of probabilities is at bottom nothing but common sense reduced to calculus"

Pierre Simon de Laplace

Philosophical essay on probabilities

Counting





# Statistics

?/¿

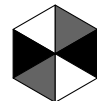
Statistics are used to handle great volumes of data, from ages and numbers of inhabitants to shopping habits. How can you represent these data in a useful way? How can you extract information from them? How can you use them to predict the future? How can you spot an anomaly?

**One of the methods you can use is to compare an observation with a random theoretical model. This is how Benford's law is used to spot cheats!**

Surveys are a branch of statistics. You handle them much like you'd taste a bowl of soup: you mix it up, you take a big enough sample... and sometimes you get it wrong. All it takes is the clove turning up in your spoonful to completely skew your impression of the soup!

!/i

# Statistics



## Sweets



**Place** one of the frames on the picture of all the sweets and **count** how many are inside it.

As the surface area contained within this frame is 200 times smaller than that of the picture, you can multiply the number you've arrived at to get an estimate of the total number of sweets in the picture.



Sweets



This counting method is used to estimate a very large number of individuals. This could be the number of demonstrators, for example, or the number of flamingos on a lake... It works well here, because the sweets cover the picture in an even way.

This is more difficult when you want to count demonstrators, animals and stars, which can cluster into groups and leave empty pockets. **How do you choose the right sample to count?**

20

## How many fish?



**Imagine** that the sphere contains fish. 400 of these are marked with green.

How many fish are there altogether in the jar?

**Turn** the sphere.

**Examine** the marbles you have "fished out".

Using this sample **estimate** the total number of marbles...



This method is used by scientists to count animal populations.

**But this is only an estimate!**

If you do this experiment again, you'll see a different number of marbles in the sample. Hence the famous margin of error, which all honest surveys should include.



How many fish?

## A fair sample



How many blue characters are there?

Instead of counting them all, make a survey. **Take** a sample: **place** one of the two grids over the characters and **count** those that are visible. **Start again** with the other grid.

Do both samples seem to be of the same quality?



A fair sample must be taken from a properly mixed population.

Ask people in a provincial marketplace about their political views on a weekday morning and then ask people in a Parisian café on a Saturday night the same question. The results won't be the same!

Here, only one of the grids represents a fair sample.



A fair sample

21

## All aboard the curve!



Are you an adult?

Stick a coloured sticker after the others on the line corresponding to your height.

Are you under 18?

Place it on the column corresponding to your age.

Observe the result we've got since the exhibition first opened (the 6<sup>th</sup> of December 2016)...

What can you see?



The results for the under-18s should form a growth curve, the same as you get in a child health care record book.

And for the adults' results, this should form a "bell curve", or Gaussian function: lots of results around the average height, not so many very tall or short. The genetic lottery explains this observation (see Galton's box).



All aboard the curve!



## Chance?

?/¿

You are confronted with a long series of digits. Have they been chosen at random?

If so, the digits should each occur at around the same frequency. But that's not enough: the series "012345678901234567890..." has nothing random about it!

**Let's look a little deeper:** pairs of digits, from 00 to 99, should also occur with the same frequency. It's the same for sets of three digits, four, five, and so on.

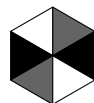
But the series "012345678910111213141516171819202122..." is not at all random, but passes these tests with flying colours!

**What does this tell you?**

That it's not so easy to flush out a "false random".

!/i

Chance?



## The rain wheel



Spin the wheel.

By falling at random, the marbles make a noise like that of light rain. This noise is similar to that made by “rain sticks”, long tubes filled with little stones.

*In desert regions in the north of Chile, during rain ceremonies, native people use sticks made from dried cactus branches to imitate this sound.*



What happens when you superimpose a large number of random signals (in this case, falling marbles)?

The result is meaningless, something we call “white noise”, made up of a random mix of every different sound frequency. The visual equivalent is the “snow” you get on an old TV which isn’t receiving any signal.



The rain wheel

23

## The symphonic poem, György Ligeti, 1962



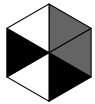
At the beginning of this video, there seems to be no order whatsoever. You cannot hear the regular ticking of each metronome at all.

**Surprisingly:** you’d even think that these were completely random beats! When there are almost none left, you can finally make out the regular rhythm of each one (the last three metronomes beat out the rhythm of church bells!).

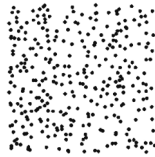
Once again, proof that perfect order can look a lot like random chance!



The symphonic poem, György Ligeti, 1962



Take a handful of grains of rice and try to distribute them “randomly”. By trying to place them everywhere, you’ll end up with something that looks a little like this:

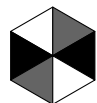


Now drop them completely at random. The result will be closer to this:



The first result is much too regular. It looks like well-raked gravel. In reality, random chance produces “clusters”. This is the same when looking at results over time, too: when you toss a coin, you have a one-in-two chance of it landing heads. But it would look very strange indeed to throw heads, tails, heads, tails, heads, tails! Similarly, the once-in-a-century flooding of the Seine in Paris arrives on average once a century, not exactly every hundred years.

So there is no “law of series”. It is perfectly normal to regularly observe a series of disasters... that don’t necessarily have any common link.





## Red and blue at random



Try and form a random sequence of two colours.

**Hint:** don't change the colours too rarely, or too often!

**Do you think you're good at creating randomness?**

### Difficult to imitate randomness...

Did you know that in a random sequence the probability of having at least one series of five red items or one series of five blue items is about 80%?

Or that there is almost a 90% chance of not having exactly 25 red items and 25 blue items?

**Our intuitive perception of randomness hardly includes such imbalances!**



Red and blue at random

25

## Identical?



On each of the 6 wheels are 16 different coloured shapes. **Spin** the wheel and **wait** for them all to stop moving.

**Look at** the 6 symbols lined up under the black line: **do you** see several identical shapes with the same colour?

**Repeat the experiment.**



**Let's calculate** the probability that all of the symbols are different: the second wheel must stop on one of 15 different symbols on the first, the third on one of the remaining 14, and so on.

**The calculation is, therefore,  $15 \times 14 \times 13 \times 12 \times 11 / 16^5$ , and it is irrevocable: almost two times out of three, at least two of the three symbols will be identical!**

*This is the same idea as the "birthday paradox": the probability that, in a group of 23 people, two people will have the same birthday is higher than 50%.*



Identical?

In this box, there are 10 black socks and 10 blue socks.

**How many** would you have to take out to be sure of having a matching pair? And an unmatched pair?


**Think about** the “pigeonhole principle”: if there are more pigeons to put away than there are pigeonholes, then at least one of these will contain more than one pigeon...



## The quest for socks

## The quest for socks



 The pigeonholes here are the two colours: from the third sock pulled out, one of the two will have at least two socks...

**But you’d need to pull out 11 to be certain of having two different socks!**

The pigeonholes here are socks of the same colour.

Using other pigeonholes, we know that there are at least 2 Parisians who have exactly the same number of hairs on their head!

26

**Randomly select** 23 birthdays, using the wheels.

Did you end up with at least two birthdays which were the same?


What is the probability that this will occur?



## Birthdays

## Birthdays



 Make sure you answer the right question here!

While it’s highly unlikely that, in a group of 23 people, one of them will have the same birthday as you, the probability of any two people having the same birthday is a little **more than one in two**.

And it’s almost a **nine-out-of-ten** chance for a group of **40 people!**

## Who wants to win a car?



This game needs two participants: a host and a contestant.

1. The **host** hides a prize car behind one of the three doors.

2. The **contestant** chooses a door. The host opens a different door, one which does not conceal the car.

The contestant can either stay with their original choice or change door.

What should you do?



**Surely it doesn't matter – there are two doors, so there's a one-in-two chance, right?**

**No, actually!**

Decide to change: if your first choice was right (which there is a one-in-three chance of occurring), you'll lose.

But if it was wrong (which happens two thirds of the time), the host doesn't have a choice about which one to open, so has indirectly shown you the right door!



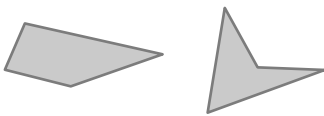
Who wants to win a car?

27

## A little order over here



**Shake** the box. Among the 5 counters, can you see 4 which form the corners of a convex quadrilateral (a shape whose angles only point outwards)?



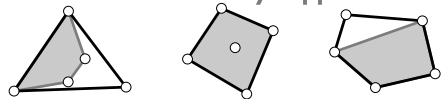
© Guillaume Reuiller/Universcience

The quadrilateral on the left is convex, the one on the right is not.

Repeat the experiment... Is this very frequent?



Five non-aligned points will always form at least one convex quadrilateral! Wrap an elastic band tight around the 5 points: it will form a 3-, 4- or 5-sided shape. And from this, you'll always be able to create the quadrilateral you're looking for. **Even in a random world, a little order will always appear.**



The black line represents an elastic band wrapped around the 5 points. The grey line shows the convex quadrilateral you want.



A little order over here

## The bees' challenge



At random, **cover** this board with orange and blue counters.

Is there a blue pathway crossing the board from east to west?

What is the probability that this will occur?



You can check this: the existence of such a path would make it impossible to have an orange path running from north to south. And inversely, if it is not there, then there must be an orange pathway.

The response is simple. If there are the same number of counters for both colours, then the chance of a path existing is exactly one in two!

*This type of problem can be used to model percolation phenomena (passing through a porous environment).*



## The bees' challenge

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Curiosities

"Imagine training a million monkeys to type randomly at a typewriter and [...] that these monkey typists worked hard for ten hours a day on a million typewriters [...]."

Curiosities



Curiosities

After working for a year, their work would contain the exact copies of every kind of book, in every language, kept in the finest libraries in the world."

Émile Borel,  
"Mechanical Statistics and Irreversibility",  
Journal de Physique,  
1913

Curiosities





# Chaos theory

?/¿

**Can you predict the trajectory of a pinball as it comes off a flipper?**

In practice, it's impossible. But in theory? It can actually be calculated quite well. However, an infinitesimal variation in angle or position will, after a few ricochets, lead to an extremely different trajectory. To predict this, you need to know the initial conditions with infinite accuracy, which is impossible.

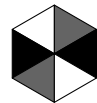
**This "extreme sensitivity to initial conditions" is characteristic of "chaotic" phenomena.**

Now we have probabilities or statistics: the trajectory of a ball on the snooker table below cannot be predicted, but the image obtained by modifying the initial conditions will be very close.

Similarly, it is impossible to predict the weather over a medium term, but we can try to predict what the climate will be like a hundred years from now!

!/i

# Chaos theory



## Chaotic double pendulums



Using the little buttons on the side, **bring** the two pendulums to the height you want. **Now let them swing!**

Both pendulums are completely identical.

**Do they start swinging the same way? And after a minute or two?**



A very small difference in speed or position quickly becomes bigger and bigger, then enormous, getting to the point where the two trajectories seem to be completely different!

**It is impossible to get the same result twice with these chaotic pendulums.**

*Double pendulums are a simple example of "heightened sensitivity to initial conditions".*



## Chaotic double pendulums

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## Pinball



**Place** the ball on the launcher, pull the bolt and release it.

Do you think you can send the ball into the same box twice in a row?

**Can you make it follow an identical (or almost identical) route twice in a row?**



Using a virtual pinball table, like the one at the post nearby, it would be possible to obtain two identical trajectories. However, on a real one, you can never perfectly reproduce the same path. What's more, every time the ball hits an obstacle, a small initial difference is magnified...

**Very quickly, two almost identical trajectories become completely different!**

*This experiment illustrates the "heightened sensitivity to initial conditions".*



## Pinball

## Virtual pinball



**Make** your own pinball table by choosing the radius and location of the three circles, and the position and launch direction for the ball. You'll only see the trajectory it takes.

**Place** the launch point inside a circle. **What happens?**

**Slide** a little piece of another disc towards the launch point.

**What happens to the trajectory? And what about if you move the launch point a little?**



If the launch point is inside a disc, the trajectory is regular and isn't affected too much by the initial conditions. But when you slide another disc inside the first, the trajectory is still predictable but becomes much more complex. A tiny variation of the initial conditions can have an enormous impact.

**In practice, you cannot predict the path it will take. The situation becomes chaotic.**



Virtual pinball

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## The virtual pendulum



**Choose** the length of both pieces of the pendulum, and the intensity and direction on the gravitational force.

**Can you make it so the movement is regular, and not chaotic?**



The easiest way to do this is to remove the second piece by placing the red and green points in exactly the same place.

You create a single pendulum with a completely regular motion: periodical swings or a full circle, depending on the gravitational force. A small disturbance won't make much of a difference.

**But of course, it's a lot more interesting to make it more complicated!**



The virtual pendulum

## Curiosities

- > Red and blue at random
- > Identical?
- > The quest for socks
- > Birthdays
- > Who wants to win a car?
- > A little order over here
- > The diamond lottery

## Probabilities

- > The second always comes first
- > The dice snake
- > Red dice
- > 1 in 10,000
- > Shapes in the fog
- > Hidden message

## > Roulette

- > The number collector
- > Loaded dice
- > Minipoly
- > Not the same colours!
- > Sicherman dice
- > Blue, yellow, green

## Chaos theory

- > Chaotic double pendulums
- > Pinball
- > Virtual pinball
- > The virtual pendulum

## Statistics

- > Sweets
- > How many fish?
- > A fair sample
- > All aboard the curve!
  
- > The bees' challenge

## Counting

- > One hundred thousand billion poems
- > Let's play some music
- > Pascal's triangle

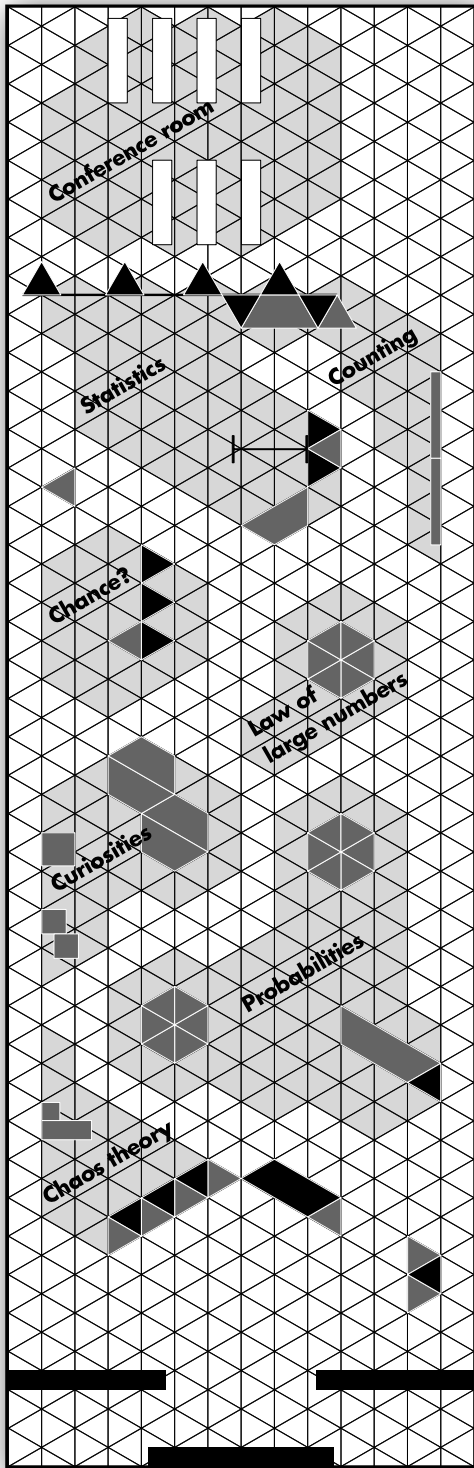
## Chance?

- > The rain wheel
- > The symphonic poem

## Law of large numbers

- > Break the code!
- > Galton's box
- > Numbers on the front page
- > The Monte Carlo method
- > Two Galton's boxes in a row







# Mathematikum

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In 2002, at the inauguration of the **Mathematikum** de Gießen, the president of Germany declared:

“Maths can actually be fun. I only realised that here.”

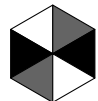
Ever since, this interactive museum has enjoyed a roaring success, with **150,000** visitors per year. **Giant soap bubbles, puzzles, experiments... and you're allowed to touch everything!** Visitors are encouraged to “**experiment, think and learn for themselves**”. The museum covers **12,000m<sup>2</sup>** and offers **170** fun exhibits for both parents and children, whether you love maths or hate it, and gives you the opportunity to get to grips with this enchanting yet ancient science.

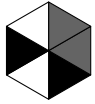
This success story began in 1994 with a travelling exhibition which is still wandering today. **It is kept up to date by the maths students at the University of Gießen.**

In 2014, the Mathematikum opened the travelling “**Random Bits**” exhibition which you can see before you. It has been adapted and enriched by the **Palais de la Découverte.**

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# Mathematikum





## The science history museum in Geneva

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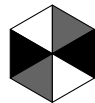
The only one of its kind in Switzerland, **the science history museum** houses a collection of historic scientific instruments used by Geneva's thinkers from the 17<sup>th</sup> to the 19<sup>th</sup> centuries (Saussure, Pictet de la Rive, Colladon, etc.). These objects have seen the development and evolution of ideas and knowledge in Geneva throughout history. They give us a better understanding of how disciplines and techniques like **astronomy, microscopes, gnomonics, electricity and meteorology** came about and developed.

Founded by a group of enthusiasts and inaugurated in 1964, the museum has been part of the natural history museum since 2015. It is housed in the Villa Bartholoni, a neoclassical jewel from 1830, located in the Perle-du-Lac Park by Lake Geneva.

Six of the original presentations from the museum are on display here. They were developed in 2012 alongside the **mathematics department at the University of Geneva** for the exhibition titled "Place your bets! Chance and probability".

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## The science history museum in Geneva





## Games of chance and addiction

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Luck and chance are things we can't control. This is why games that feature them are so captivating.

Whether you've got a **one** in **ten**, one in one **hundred** or one in a **million** chance of winning, the suspense is what keeps you coming back! But you've got to be careful and resist the impulse to keep trying to push your luck. In the end, chance will always be the master of the game.



**Gambling, especially with money, must remain a fun, occasional hobby. Don't be tempted to take it further.**

Addiction to gambling is a major hazard. This is why professionals, charities and local authorities set up policies to prevent addiction and offer support to people for whom chance has become something much more dangerous.

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## Games of chance and addiction

